International Journal of Applied Engineering Research ISSN 0973-4562 Volume 11, Number 22 (2016) pp. 10878-10886

Optimization Algorithm of Parameters of Low Head Microhydropower Plant at an Early Design Stage

A.G. Parygin , A.V. Volkov, A.V. Ryzhenkov, A.V. Naumov and A.A. Druzhinin

National Research University "Moscow Power Engineering Institute"
Krasnokazarmennaya 14, Moscow, 111250 Russia.

Abstract
The task is set to develop an original algorithm for optimization of parameters of low-head micro-hydropower plant at an early design stage for plants based on using a penstock with a turbine inside. Criteria of optimality of the full head of turbine, the flow velocity in the penstock, the reduced flow rate through the turbine are proposed. Besides, based on the aerodynamic grids theory, an algorithm is proposed for optimization of the key parameters of the blade system of the propeller turbine impeller, including an algorithm for maximization of hydraulic efficiency. Results of experimental verification of the proposed algorithm by means of a full-size model of a siphon micro-hydropower plant designed for output electric capacity 1.5kW, at the available pipeline head 2 m are presented. It is experimentally shown that the proposed algorithm of optimization of parameters gives good practical results. The quantitative analysis of the physical experiment results has shown that the calculations performed according to the proposed algorithm are of acceptable accuracy. In terms of the level of the model's output capacity, the deviation of the designed value from the one obtained experimentally was +7%. The designed value of power utilization factor was K_N = 0.339, the maximum theoretically achievable level being 0.3849. An approximate estimation of the turbine's hydraulic efficiency gave a deviation +1%. At the same time, during the experiment, the full turbine head (K_H=2/3), i.e., the optimum model operation mode, was maintained to the practical accuracy. Conclusions on optimality criteria of head and flow rate were made irrespective of the type of the turbine and may therefore be applied to all the *Kaplan*, *Francis* and *Darrieux* turbines, as well as those derived therefrom.

Keywords: micro-hydropower plant, low-head micro-hydropower plant, axial hydraulic turbine, propeller turbine, pressure pipeline, hydraulic losses, energy efficiency.

INTRODUCTION

In terms of their specific cost, hydraulic power devices are superior compared to all the other energy converters. The potential of the world's small hydropower is great and makes about 35% of the overall hydropower potential [1]. The largest share of this potential is accounted for by sources with low available heads (1 to 3 meters) [2]. It is expedient to use the potential of low-head hydropower mostly be means of micro-hydropower plants (micro-HPP). However, implementation of low-head micro-HPP involves a number of problems. First of all, they have a low available head, which reduces the capacity of the HPP and increases requirements to its energy efficiency. Besides, small power of the HPP necessitates reducing capital investments and choosing cheaper equipment [3]. Therefore, the most feasible option is to use pressure pipelines and a propeller (axial type) turbine without inlet guide vanes in low-head micro-HPP [4]. Implemented examples of such micro-HPP may be found in [3-8]. However, as of today, there exist no reliable algorithms for optimization of micro-HPP parameters at an early design stage.

MICRO-HPP PARAMETERS OPTIMIZATION ALGORITHM

1. Optimization algorithm of the turbine key designed parameters

Energy efficiency of any hydropower plant is determined by the share of hydraulic energy it can take from the stream of water. The turbine is the main element of a micro-HPP. An unconditional criterion of energy efficiency of the turbine is optimality of its designed parameters, which is preferable to be ensured as soon as possible at the first stage of the turbine creation. If the micro-HPP turbine is supposed to be placed into a penstock converting the potential energy of the flow into the kinetic energy, efficiency of the micro-HPP may be assessed with a certain criterion. The authors suggest assessing the energy efficiency of the hydraulic turbine within the micro-HPP based on energy utilization factor K_N proposed by Parygin A.G. in [9] and equal to the relation between the turbine power to the available hydraulic power of the pipeline:

$$K_N = N_T/N_P = \frac{\rho \cdot g \cdot Q_T \cdot H_T}{\rho \cdot g \cdot Q_P \cdot H_P}, \tag{1}$$

where: N_T is turbine power, Q_T is volumetric water flow rate through the turbine, H_T is theoretical head of turbine, N_P is available power of the flow in the penstock with available head H_P and volumetric water flow rate Q_P, ρ is water density, g is gravity acceleration. The K_N function unambiguously reflects the level of energy efficiency of the micro-HPP, as it numerically characterizes the share of available hydraulic energy of the pipeline utilized by the turbine. Equation (1) may be reduced to the form (2), if flow rates Q are determined in any turbine-free section of the pipeline

$$K_N = N_T/N_P = \frac{V \cdot H}{V_P \cdot H_P} = \frac{V}{V_P} \cdot K_H \cdot \eta_h, \tag{2}$$

where: V is flow velocity in water pipeline with the turbine in operation, η_h is hydraulic efficiency of turbine, V_P is flow velocity in the pipeline without turbine, K_H is factor of utilization of head in the turbine as a relation between full turbine head $H=H_T/\eta_h$ to available pipeline head H_P. The ratio of velocities in equation (2) may be determined through heads in the pipeline:

$$\frac{V}{V_P} = \frac{\varphi \cdot \sqrt{2 \cdot g \cdot (H_P - H)}}{\varphi_P \cdot \sqrt{2 \cdot g \cdot H_P}}, \tag{3}$$

Velocity factors φ are determined by equations (4), in which ξ are hydraulic friction factors in the Darcy-Weisbach equation [10]:

$$\varphi = (1 + \xi)^{-0.5}\ ; \qquad \varphi_P = (1 + \xi_P)^{-0.5}. \tag{4}$$

In the general case, the values of ξ and ξ_P are not equal, as flow velocities V and V_P are not equal, and the Reynolds numbers in a pipeline with a turbine and without the same are different. However, with available heads of at least 2 meters, the Reynolds numbers are within the self-similarity zone, or at its boundary [11]. This makes possible equating velocity factors φ and φ_P. With this assumption, equation (2) will look as (5):

$$K_N = \sqrt{(1 - K_H)} \cdot K_H \cdot \eta_h. \tag{5}$$

For a perfect turbine with hydraulic efficiency equal to 1, the function $K_N(K_H)$ has a form presented in figure 1.

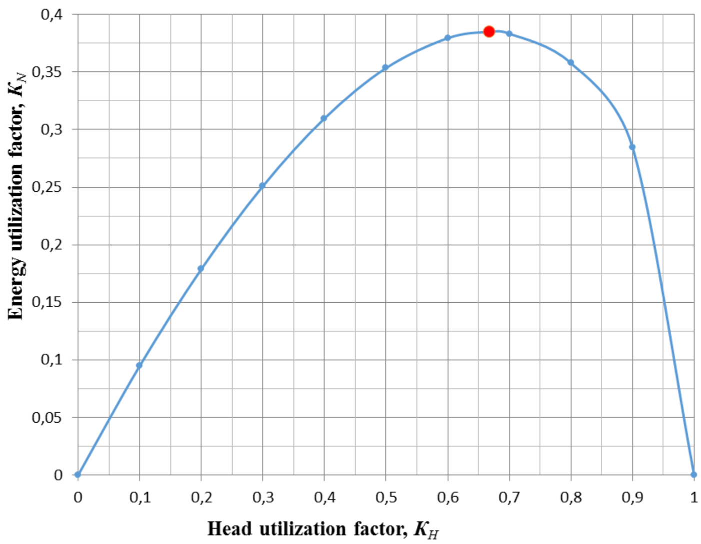


Figure 1. Graphic interpretation of function $K_N = f(K_H)$ for a perfect hydraulic machine.

The graph on figure 1 shows availability of the maximum of function $K_N(K_H)$, which gives the optimum relation between the full head on the turbine and the available head of the pipeline, i.e., the value of $(K_H)_{opt}$., which would be the best in terms of energy efficiency. To determine this value, we just have to equate the derivative of K_N to K_H to zero:

$$\frac{\partial K_N}{\partial K_H} = \frac{\partial}{\partial K_H}\left(K_H \cdot \sqrt{1 - K_H}\right) \cdot \eta_h = \frac{2 - 3K_H}{2\sqrt{1 - K_H}} \cdot \eta_h = 0. \tag{6}$$

Therefore, the maximum of function $(K_N)_{max} = \frac{2 \cdot \eta_h}{\sqrt{27}} \approx 0{,}3849 \cdot \eta_h$ is achieved with K_H=2/3. The result obtained enables to conclude that the HPP with a turbine located in the penstock will have the highest energy efficiency if the head utilized by the turbine is exactly 2/3 of the available head of the pipeline, i.e., $(H)_{opt}=2/3 \cdot H_P$.

Now that we know the value of the optimum head on the turbine $(H)_{opt}$, we can determine the optimum velocity in the pipeline:

$$(V)^2_{opt} = \frac{2 \cdot g \cdot H_P \cdot [1 - (K_H)_{opt}]}{1 + \xi} = \frac{2 \cdot g \cdot H_P}{3 \cdot (1 + \xi)}, \tag{7}$$

Equation (7) enables to determine a parameter essential for the turbine, namely, the optimum reduced water flow rate [12]:

$$(Q_{11})_{opt} = \frac{Q}{D_1^2 \cdot \sqrt{(H)_{opt}}} = \frac{(V)_{opt} \cdot S}{D_1^2 \cdot \sqrt{(H)_{opt}}} = \frac{\pi}{4} \cdot \sqrt{\frac{g}{(1+\xi)}}, \tag{8}$$

which makes it possible to find the outer diameter D_1 of the turbine impeller. Here, a curious circumstance should be noted, which follows from equation (8): the value of $(Q_{11})_{opt}$ only depends on hydraulic friction of the pipeline and is not related to the turbine, as such. This relationship is presented in figure 2.

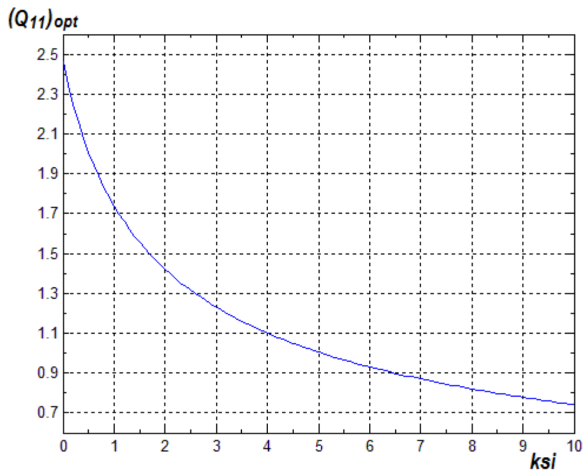
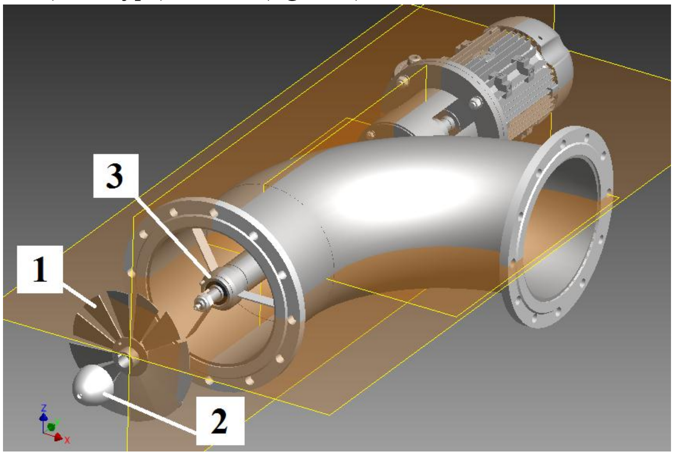
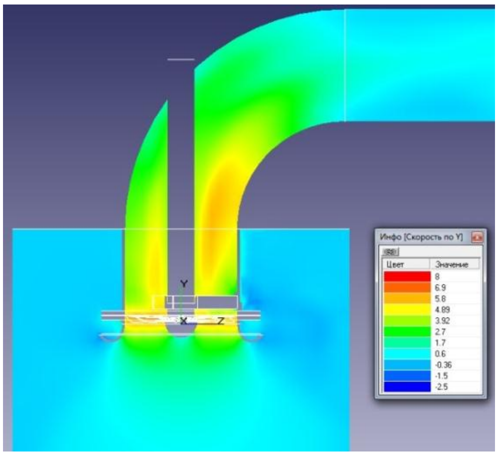


Figure 2. Dependence of the optimum reduced flow rate of water through the turbine on hydraulic friction of the pipeline

The problem of estimation of hydraulic friction of the HPP pipeline is complicated by the fact that this accommodates the turbine shaft elements, which hydraulic friction is a priori unknown. In this case, it could be useful to resort to 3D-simulation of the pipeline and to conduct a virtual experiment in the CFD-code environment (in particular, this could be ANSYS or FlowVision). As an example, the authors present the general view of such model for a version of hydraulic unit with a propeller (axial type) turbine (figure 3). The results of the virtual experiment for the minimum hydraulic diameter of the pipeline equal to 0.2604 m in the range of pipeline flow velocities from 1.25 m/sec to 7 m/sec are presented in the table below and in figure 4.



a) general view of hydraulic unit model.



b) diagram of flow velocity distribution in the penstock model.

Figure 3. A flow part fragment of designed micro-HPP pipeline of 3D-model.

Table 1. Designed values of hydraulic friction factor ξ

Flow velocity, *V* m/sec	**Value of factor ξ**
1.25	14.152
1.5	9.932
1.75	7.419
2.0	5.528
2.5	3.538
3.0	2.328
4.0	1.435
5.0	0.905
6.0	0.653
7.0	0.48

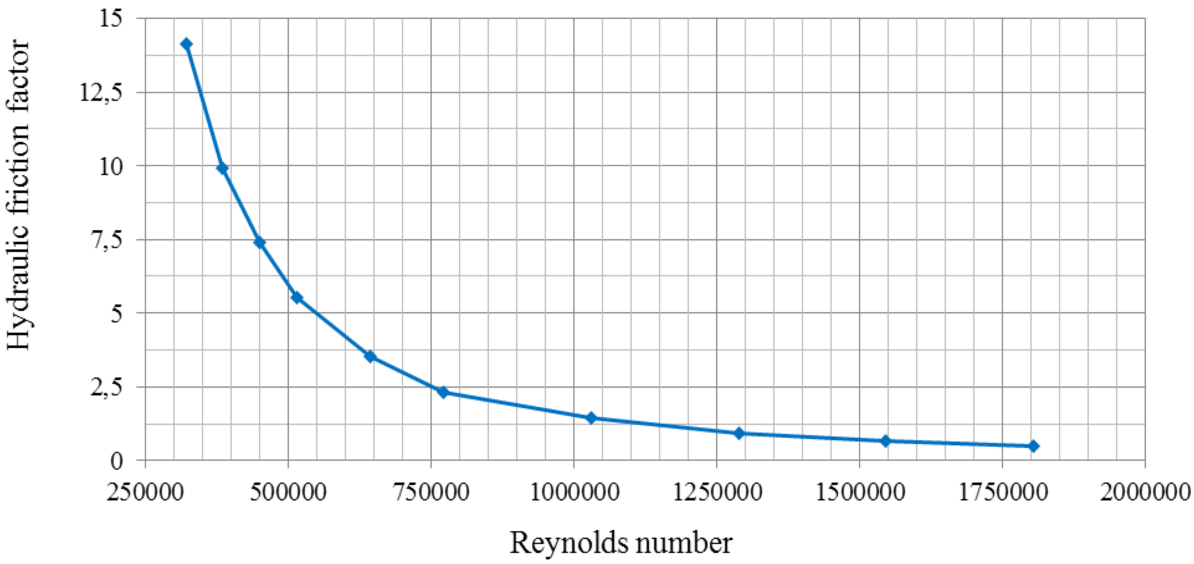


Figure 4. Virtual experiment results for hydraulic friction factor determination of the pipeline flow part.

In the virtual experiment, a model of a non-compressible liquid was used, which enables to calculate the velocity and the turbulence of the flow through supplementing the Navier-Stokes equation with additional components describing turbulent viscosity and turbulent thermal conductivity. Besides, a *k*-ε turbulence model was used for the logarithmic law of distribution of velocity gradient in the boundary layer. A computational grid created for the virtual experiment contained 512,000 cells. The simulation enabled to calculate full pressure losses based on an integral parameter as the difference between the values of full pressure in the inlet and outlet sections of the pipeline. The values of function $\xi(V)$ were determined with the Darcy-Weisbach equation (Table 1). Here, it should be noted that the already known function $\xi(V)$, as such, does not enable to determine the value of the value of the optimum flow velocity in the pipeline; this would require solving numerically a system made up of numerical (experimental) interpretation of function $\xi(V)$ and equation (7), with values of $(V)_{opt}$ set successively until obtaining the acceptable accuracy of the result.

2. Algorithm for maximization of turbine efficiency in the designed mode

Optimization of the turbine parameters does not only mean selection of the designed head and diameter of the impeller, but also maximization of its efficiency at given operating conditions in the pipeline. Most likely, finding an analytic solution for the problem of the turbine efficiency optimization to the full extent will be impossible in the foreseeable future. However, it is possible to get as close to such solution as possible for an elemental version of the propeller turbine, if we have the problem slightly simplified, effects of the impeller's variable parameters on variation of secondary hydraulic losses in the turbine not taken into account. The aforesaid losses are meant to include hub losses, blade tip-leakage flow losses, losses from asymmetrical flow in the turbine stator, and other losses associated with its stator and hub one way or another. If we assume a weak dependence of these losses on designed parameters of the impeller (which is quite legitimate), those may be "denoted" as losses in the pipeline and taken into account by increasing factor ξ in equations (7) and (8) by the value of secondary losses in the turbine. Then, further, we would be able to be limited to consideration of the vane system's hydraulic friction, which, in this context, will be hydraulic efficiency η_h, used in equations (2), (5) and (6). With the above assumptions made, the vane, as such, may be regarded as having an infinite length, i.e., may be replaced with a profile of a span equal to 1 located on the mid-radius of the impeller. Figure 5 presents a designed diagram of the vane profile, the layout of velocities and the diagram of forces acting on the profile moving progressively across the axis of the flow of liquid viscous medium.

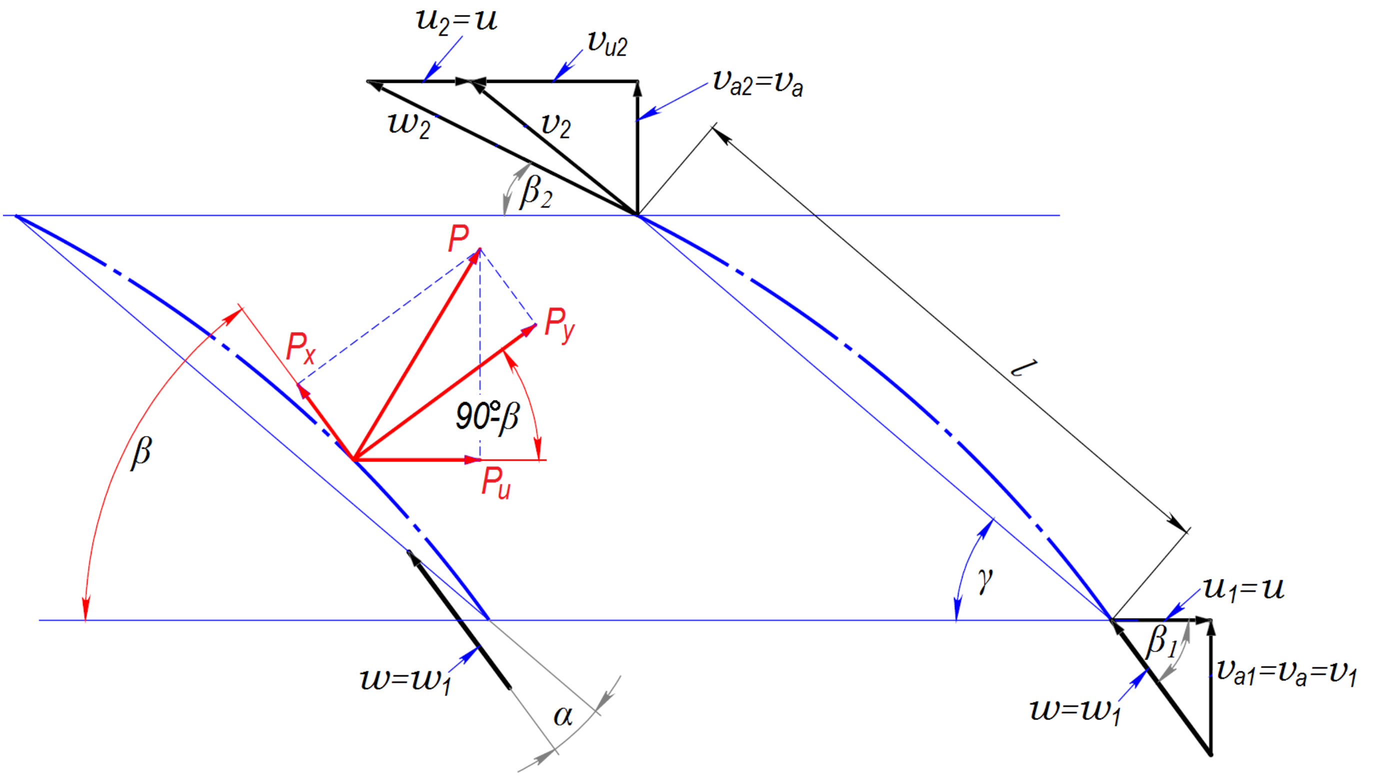


Figure 5. Designed diagram of the blade motion of the turbine impeller with a cylinder-shaped hub (the profile of the blade is conventionally presented with its mean line only).

In figure 5:
l is length of chord of profile;
γ is angle of setting of profile;
u is linear (circumferential) velocity of the blade motion along mid-radius *r* of the impeller ($u=\omega\times r$, where ω is angular speed of impeller rotation);

v is absolute flow velocity;
v_a is axial flow velocity;
v_u is circumferential component of absolute flow velocity;
w is relative flow velocity;
w_u is circumferential component of relative velocity;
$\beta_1=\beta$ is striking angle;
$\beta_2<\beta$ is runoff angle;
α is profile attack angle;
P_y is lifting force of profile;
P_x is drag of profile;
P is the force acting on the profile from the flow;
P_u is circumferential component of force P.

According to Zhukovskiy, for a profile with a span equal to 1 [13]

$$P_y = C_y \cdot \rho \cdot \frac{w^2}{2} \cdot l \qquad \text{and} \qquad P_x = C_x \cdot \rho \cdot \frac{w^2}{2} \cdot l, \tag{9}$$

where C_y is lifting force factor, and C_x is drag factor. For impeller vanes, as wings of finite span Δr, equations (9) may be converted into:

$$P_y = C_y \cdot \rho \cdot \frac{w^2}{2} \cdot S \qquad \text{and} \qquad P_x = C_x \cdot \rho \cdot \frac{w^2}{2} \cdot S, \tag{10}$$

where $S = l \cdot \Delta r \cdot z$ is an aggregate area of all the vanes, and z is their number in the impeller.
As follows from figure 5, the circumferential component of force P summed up for all the blades may be presented through P_y and P_x by the following relation:

$$P_u = P_y \cdot \cos(90 - \beta) - P_x \cdot \cos\beta = P_y \cdot \sin\beta - P_x \cdot \cos\beta = \\ = (C_y \cdot \sin\beta - C_x \cdot \cos\beta) \cdot \rho \cdot \frac{w^2}{2} \cdot S. \tag{11}$$

The torque of force P_u relative to the impeller axis (the turbine rotor rotation axis) may be presented as a product of force P_u and its arm, which is mid-radius r of the impeller. On the other hand, the same torque may also be expressed through useful power N on the turbine shaft and its rotor rotation frequency ω [14], i.e.:

$$M_{KP} = P_u \cdot r = \frac{N}{\omega} \qquad \text{or} \qquad P_u = \frac{N}{\omega \cdot r} = \frac{N}{u}. \tag{12}$$

With consideration of (10), the previous equation could be easily used to obtain:

$$\left[\rho \cdot \frac{w^2}{2} \cdot S\right] = \frac{N}{u} \cdot \left(\frac{1}{C_y \cdot \sin\beta - C_x \cdot \cos\beta}\right), \tag{13}$$

which will be useful further on.

Now, we can proceed to assessment of hydraulic efficiency η_h of the vane system. Its value is determined by a relation between turbine theoretical head H_T and head waste H^* in the blade system:

$$\eta_\Gamma = \frac{H_T}{H_T + H^*} = \frac{\frac{H_T}{H^*}}{\frac{H_T}{H^*} + 1}. \tag{14}$$

As follows from the Euler equation

$$H_T = \frac{N}{g \cdot G} = \frac{N}{g \cdot \rho \cdot v_a \cdot S_T}, \tag{15}$$

where S_T is area of the flow part of the impeller in a section normal to the turbine axis. Head waste H^* caused by drag of the vane system is determined by static pressure losses Δp in the flow:

$$H^* = \frac{\Delta p}{g \cdot \rho} = \frac{P_x}{g \cdot \rho \cdot S_T \cdot \sin\beta}, \tag{16}$$

where $[S_T \cdot \sin\beta]$ is area of the flow part of the impeller in a section normal to the vector of relative velocity w (see figure 5).
Given (15), (16) and further (10), the relation between H_T and H^*, which makes part of equation (14) and totally determines the value of η_h may be brought to the following form:

$$\frac{H_T}{H^*} = \frac{N \cdot \sin\beta}{P_x \cdot v_a} = \frac{N \cdot \sin\beta}{C_x \cdot v_a} \cdot \left[\rho \cdot \frac{w^2}{2} \cdot S\right]^{-1}. \tag{17}$$

In (17), the product of arguments enclosed in square brackets corresponds to the left side of the previously obtained equation (13). Given this and the fact that, as follows from figure 4

$$\frac{u}{v_a} = \mathrm{ctg}\,\beta, \tag{18}$$

the ratio of C_y to C_x involves a notion of aerodynamic quality k of profile [15]. Then, it would be easy to obtain from equation (17), after a series of trigonometric transformations, the following equation:

$$\frac{H_T}{H^*} = \frac{u \cdot \sin\beta}{v_a \cdot C_x} \cdot \left(C_y \cdot \sin\beta - C_x \cdot \cos\beta\right) = \\ = \frac{k}{2} \cdot \sin 2\beta - (\cos\beta)^2. \tag{19}$$

Inserting (19) into (14), we can, after simple transformations, obtain a final expression of hydraulic efficiency of the vane system:

$$\eta_h = \frac{k^*}{k^*+2}, \text{ where } k^* = k \cdot \sin 2\beta - 2 \cdot (\cos\beta)^2. \tag{20}$$

Figure 6 presents results of calculation of function $\eta_h(\beta, k)$ for a discrete series of values of k characteristic for typical aerodynamic profiles.

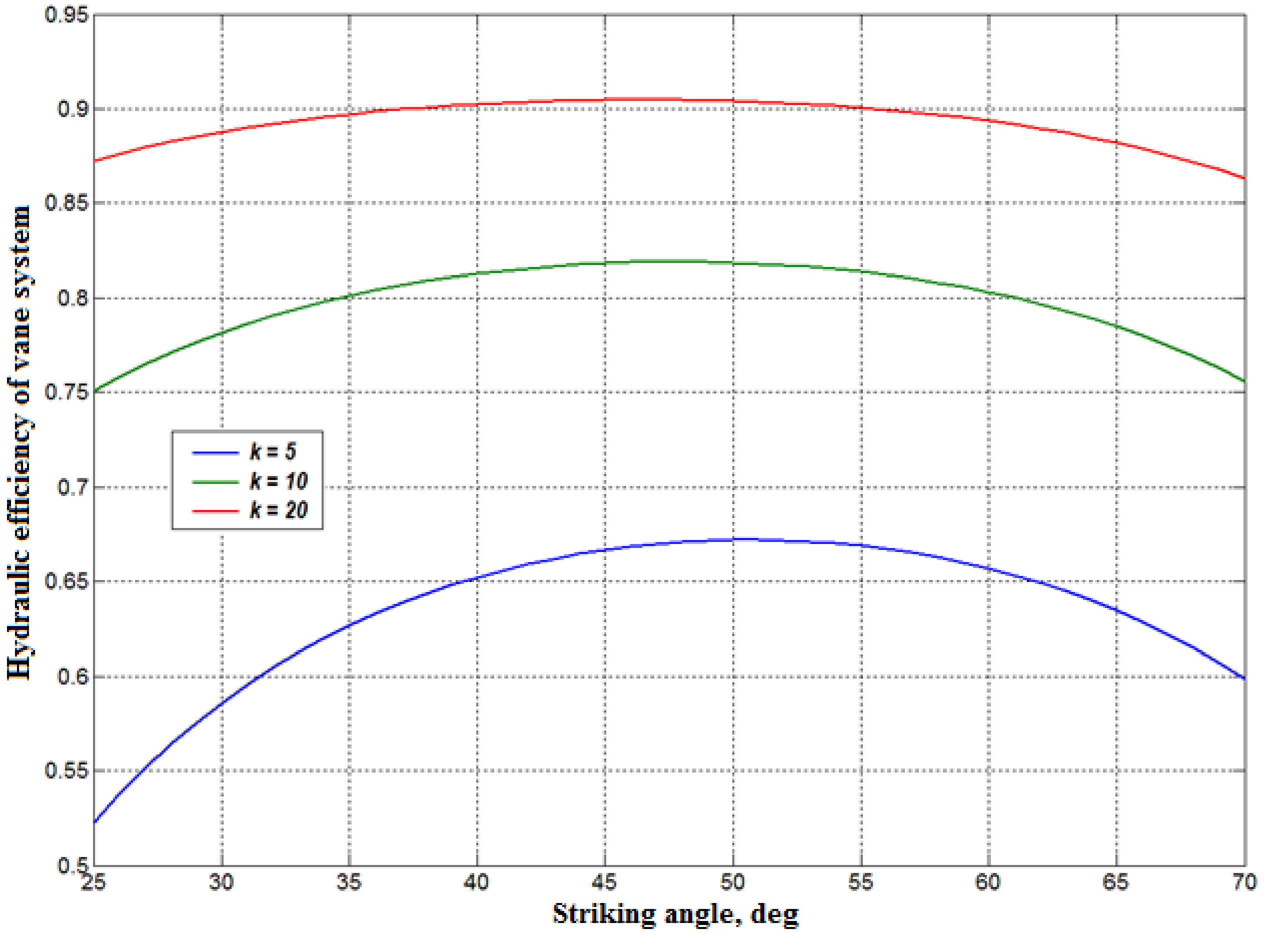


Figure 6 – Graphs of function $\eta_h(\beta, k)$.

RESULTS OF EXPERIMENTAL VERIFICATION OF OPTIMIZATION ALGORITHM

For the proposed algorithm verification of micro-HPP parameters optimization, there was developed a model of siphon micro-HPP, its hydropower unit equipped with a model of propeller turbine having no inlet guide vane system (Figure 7). The micro-HPP model was designed for operation with available head of 2 m (based on the difference of elevations between free surfaces of the upper and lower tanks of the experimental bench) (Figure 8). Therefore, the optimum full head on the turbine must, according to the algorithm, be 1.33 m. The calculations under the proposed algorithm also produced the value of $(V)_{opt}$ =3.023 m/sec and $(Q)_{opt}$ =2.05 m^3/sec at ξ=0.438. The authors set a task to obtain at the output of the generator the electric power of at least 1500 W, with energy losses in shaft seals, bearings and the generator of about 8%. As a result, we eventually chose a turbine with the outer diameter of the impeller 0.250 m and the hub diameter 0.075 m. To make the turbine cheaper and to ensure the accuracy of calculations, as an aerodynamic profile of 12 blades, we chose a flat plate having the maximum aerodynamic quality k=10 at attack angle α=3^o [17]. It should be pointed out that in calculations of efficiency of the blade system based on equation (20), the value of the maximum profile quality was adjusted upwards with consideration of the lift factor [18], which, according to the Voznesenskiy diagram, has a value 2.4 with the chosen relative tangential blade spacing equal to 1, and the blade pitch angle 15^o. As a result, the striking angle β was chosen at 18^o, which is determined by the ratio of the axial velocity of the flow in the turbine (v_a=3.26 m/sec) to the linear velocity of turbine rotor rotation (u=10 m/sec) at mid-radius. The point is that, for a model of hydraulic unit, an asynchronous squirrel-cage machine was chosen with synchronous rotation frequency of 1,000 RPM. The choice was made deliberately to test the possibility of efficient operation of a propeller turbine at high speeds, i.e., without a speed-up unit. As a result, the blade system with the chosen parameters must, according to equation (20), have efficiency of 86%.

Figure 7. Photograph of the hydraulic unit model with a propeller turbine.

The results of measured parameters of linear filtration of the micro-HPP, considered in the dynamics of acceleration in the conditions of operation for a global network, are presented in figure 9. After switching on of the micro-HPP, a generator-motor was connected to the network, and the hydropower unit started operating in the pumping mode.

Figure 8. Photograph of the siphon penstock model fragment of the micro-HPP on the experimental bench.

Upon starting the siphon, a transient mode was observed for 0.6 sec, with the rotor accelerated to the synchronous frequency (1,000 RPM), with the motor power consumed from the network. Then, the micro-HPP started operating in the generator mode. Coming to the settled generation mode did not take more than 3 sec. The maximum power provided by the generator amounted to 1,606 W.

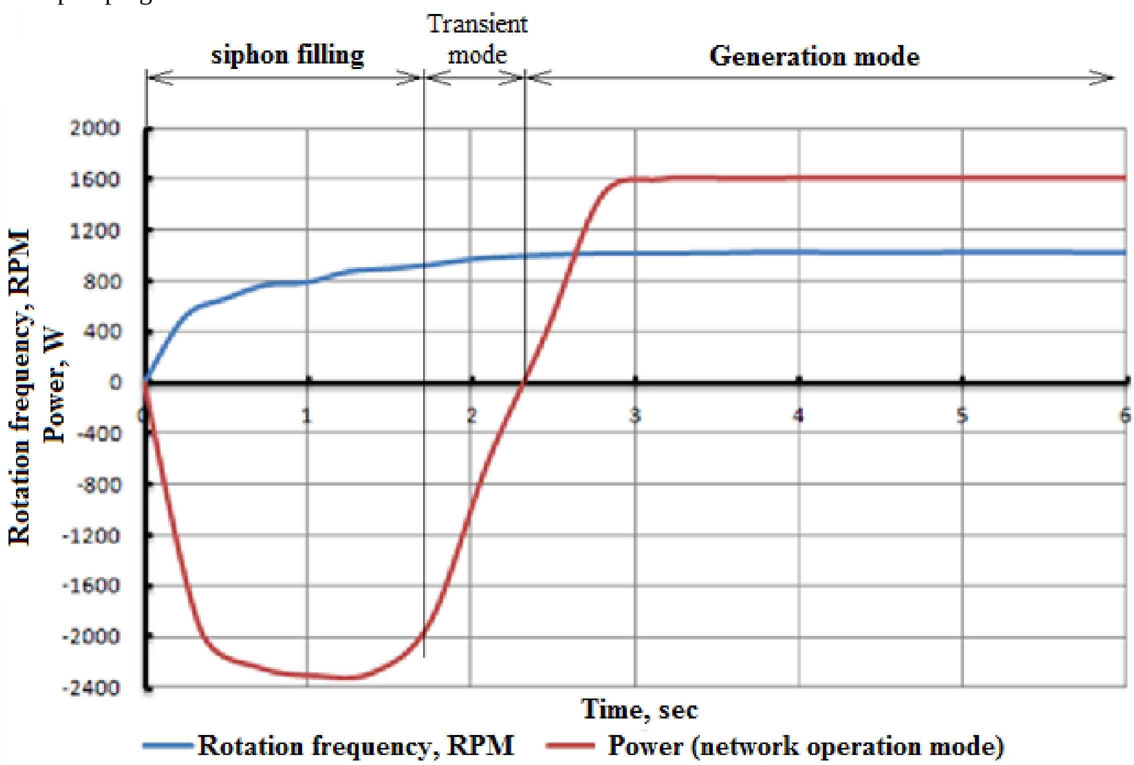


Figure 9. Parameters oscillogram of micro-HPP model.

DISCUSSION

As the quantitative analysis of results of the physical experiment with a model of siphon micro-HPP has shown, the accuracy of calculations performed under the proposed algorithm is acceptable. Thus, in terms of the level of output power of the micro-HPP, the deviation of the designed value from the one obtained in the experiment was +7%. The designed value of the power utilization factor was K_N = 0.339, the maximum theoretically achievable level being 0.3849. An approximate estimation of the turbine's hydraulic efficiency gave a deviation +1%. At the same time, during the experiment, the full turbine head (K_H=2/3), i.e., the optimum model operation mode was maintained to the practical accuracy. It should be noted that conclusions on criteria $(K_H)_{opt}$ and $(Q_{11})_{opt}$ were made irrespective of the type of the turbine and may therefore be applied to all the *Kaplan, Francis* and *Darrieux* turbines, as well as those derived therefrom.

The calculation results analysis of an axial type propeller turbine hydraulic efficiency based on the aerodynamic grids theory enables to make the following conclusions.

CONCLUSIONS

First, in the range of the striking angles β from 0° to 90°, function $\eta_h(\beta, k)$ has the maximum, which corresponds to the optimum value β_{OPT}. Here, for high aerodynamic quality of profile, β_{OPT} is tending to the value of 45°. This result correlates well with experimental results presented in [18]. On the other hand, for poor quality profiles, β_{OPT} is increasing. Second, in case of deviation of β from the optimum value by ±5°, the efficiency would fall insignificantly (within 1%). Here, the range of "acceptable" deviations of β from β_{OPT} would increase fast along with the growth of the profile quality. Third, with the optimum angle β_{OPT} tending to the value of 45°, the vector triangle (layout) of flow velocities on the mid-radius of the turbine impeller is found to be virtually isosceles. This means that, in the optimum mode of operation of the turbine (in the maximum efficiency mode), it is preferable to observe an approximate equality of axial flow velocity v_a and linear velocity u of blade system rotation, i.e., $v_a \approx u$. To this effect, one may vary the turbine rotor rotation speed. Yet, this idea needs further exploration, as alteration of the vector triangle of velocities in the turbine would bring about a change in the theoretical head and hydraulic efficiency as well.

Therefore, in the opinion of the authors of this paper, application of the proposed algorithm enables, at an early stage of creation of the micro-HPP, to take into account, to a sufficiently high accuracy, all the key interrelations of its parameters and to determine their optimum values, which do not only determine the optimum power of the turbine, but also the maximum energy efficiency of the entire micro-HPP.

ACKNOWLEDGMENTS

The study is done with financial support of the Russian Federation Ministry of Education and Science. Grant agreement #14.574.21.0076 dated June, 27 2014 (RFMEFI57414X0076 is unique identification number).

REFERENCES

[1] R. Taylor, "Hydropower". In A. C. J. Trinnaman (Ed.), Survey of Energy Resources, Oxford: Elsevier Science, pp. 199-232. http://dx.doi.org/10.1016/B978-008044410-9/50012-7

[2] S.J. Williamson et al., "Low head pico hydro turbine selection using a multi-criteria analysis". Renewable Energ, vol. 61, pp. 43-50, 2014.

[3] Manuals and Guidelines for Micro-hydropower Development in Rural Electrification. Japan International Cooperation Agency (JICA). Vol. I, June 2009.

[4] A. Date, et al., "Performance Investigation of a Simple Reaction Water Turbine for Power Generation from Low Head Micro Hydro Resources". Smart Grid and Renewable Energy, vol. 3, pp. 239-245, 2012. http://dx.doi.org/10.4236/sgre.2012.33033

[5] F. Pochylý, M. Haluza, P. Rudolf, F. Šob"Vírová turbina", , Pat. No. 292197 CZ, 2003.

[6] M. Haluza, et al. "The swirl turbine". 26th IAHR Symposium on Hydraulic Machinery and Systems. IOP Conf. Series: Earth and Environmental Science, 19–23 August 2012, Beijing, China.

[7] M. Varchola, et al. "Jednoduchá turbína na veľmi malý spád". Zborník príspevkov 22 Medzinárodná konferencia o vodnej energetike HYDROTURBO 2014. Bratislava, Slovenská republika Sept. 23-25.

[8] J. Šoukal and R. Kubíček, "Turbíny pro malé vodní elektrárny s nízkým spádem". Energetika, vol. 1 no. 65, pp. 26-32, 2015.

[9] A.G. Parygin et al., "Commentary on the efficiency of selected structural designs of low head micro hydraulic power plants". Modern Applied Science, vol. 9, no. 4, 2015. DOI: 10.5539/mas.v9n4p116.

[10] A.D. Altschul, "*Hydraulic resistance,"(Ed.2)*. Moscow: Nedra, 1982.

[11] J. Bachelor, "*An introduction to fluid dynamics" (B. Vahomchik, A. Popov Trans.).* Moscow: Regular and chaotic dynamics, 2004.

[12] L.G. Krivchenko, "*Hydraulic machines:Turbines and pumps"*. Moscow: Energoatomizdat, 1978.

[13] L.D. Landau and E.M. Lifshitz, "*Theoretical physics", T.VI: Fluid Dynamics*. Moscow: Physmathlit, 2006.

[14] A.A. Lomakin, "*Centrifugal and axial pumps"*. Leningrad: Mechanical Engineering, 1966.

[15] B.V. Ovsyannikov and B.I. Borowski, "*Theory and calculation of power supply units for liquid rocket engine" (Ed.3).* Moscow: Mechanical Engineering, 1986.

[16] V.M. Ivanov, et al., "On the question of the method of calculation of the flow part of axial turbines of low power hydropower plants". Polzunovskiy vestnik, no. 4/2, pp. 61–69, 2010.

[17] S. I. Dievnin, "Aerohydromechanics of poor-streamlined constructions". L., Sudostoenie, pp. 320, 1983.

[18] M. Varchola, et al. "Methodology of 3D hydraulic design of a impeller of axial turbo machine", *Engineering MECHANICS, Vol. 20, No. 2, p. 107–118,* 2013.